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In any case the error of draughting may or may not combine favorably with these theoretical errors, but this we cannot control and we have to leave these accidental errors out of consideration.

It should be observed that if φ is negative the errors are numerically equal but of opposite signs to those in the positive quadrant.

NOTE.—Equation (2) is strictly true, however, only if $\varphi = \frac{1}{6}\pi$ or $= \frac{1}{2}\pi$, and for the approximate construction of a polygon. For any angle φ to be multisected we have, if φ_1 is one part reckoned from B, strictly

$$\frac{x}{x_1} = \frac{\varphi}{\varphi_1} \text{ where } x = \frac{r\sqrt{3}\,.\sin\varphi}{\sqrt{3}+\cos\varphi}, \text{ and } x_1 = \frac{r\sqrt{3}.\sin(\varphi_1-\Delta)}{\sqrt{3}+\cos(\varphi_1-\Delta)}$$

and Δ is the correction to be applied to the angle $\varphi_1 - \Delta$, which the construction gives. We have then the equation:

$$0 = \varphi(\sqrt{3}+\cos\varphi)\sin(\varphi_1-\Delta)-\varphi_1\sin\varphi[\sqrt{3}+\cos(\varphi_1-\Delta)],$$

and the maximum or minimum condition is

$$0 = \varphi(\sqrt{3}+\cos\varphi)\cos(\varphi_1-\Delta)-\sin\varphi[\sqrt{3}+\cos(\varphi_1-\Delta)]+\varphi_1\sin\varphi \times\sin(\varphi_1-\Delta).$$

CORRESPONDENCE.

EDITOR ANALYST:

Since reading your note on page 150 of the ANALYST, I have made some vain essays at a demonstration of the proposition stated in Problem 211. Though I failed in my main object, some of the results I attained in regard to triangular numbers seem to me of some interest, and may appear to you of sufficient interest to warrant their insertion in your Journal.

JOHN MACNIE.

Let $N = \frac{1}{2}(n^2+n) = t_n$ be the nth triangular number, then

$$8N+1 = (2n+1)^2.$$

Hence, in order that an integer N, be a triangular number, it is necessary and sufficient that $8N+1$ be a square number. Also, any odd square number is of the form $8t_n+1$.

Ex. 1. Let $N = 55$; $8N+1 = 441 = 21^2$. $\therefore 55 = t_{10}$.

In the same way we find that for N to be the sum of two triangular numbers, t_n and t_p, it is necessary and sufficient that $8N+2 = (2n+1)^2+(2p+1)^2$, i. e., be the sum of two odd squares.

Ex. 2. Let $N = 93$; $8N+2 = 746 = 25^2+11^2$. $\therefore 93 = t_{12} + t_5$.

In order that N may be the sum of three triangular numbers, t_n, t_p, t_r, it is necessary and sufficient that $8N+3=(2n+1)^2+(2p+1)^2+(2r+1)^2$, i. e., be the sum if three odd squares.

Ex. 3. Let $N=1878$; $8N+3=15027=121^2+19^2+5^2$. $\therefore 1878 = t_{60}+t_9+t_2$.

Since every odd square number, as we have seen above, is of the form $8t_n+1$, the sum of any three odd squares must be of the form $8N+3$. But of the converse proposition, that $8N+3$ is always the sum of three odd squares, I have been unable to obtain a demonstration, though the fact can always be verified upon trial. The greatest square $(2n+1)^2$ is usually the greatest odd square in $8N+3$, or the next odd square below; the second square in the sum is, in like manner, usually the greatest odd square in $8N+3-(2n+1)^2$.

Perhaps some reader of the Analyst can find a demonstration of one or both of the following propositions, in which N, x, y, z, may be integers or zero.

$$8N+3=(2x+1)^2+(2y+1)^2+(2z+1)^2. \qquad (1)$$

$$4N+1=(2x+1)^2+4y^2 \qquad +4z^2. \qquad (2)$$

Note on the Solution of Prob. 217, by the Editor.—When the note at the foot of page 155 was written, we supposed that Prof. Johnson's solution necessarily gave one branch of the curve on each side of the axis of x, as it was supposed that the triangles were only drawn on *one side* of the hypothenuse; and the integration we gave at page 156 was made with that understanding.

By drawing the triangles on *both sides* of the hypothenuse, and describing squares on the *outside* of the legs *above* the hypothenuse, and on the *inside* of the legs *below* the hypothenuse, both branches of the locus will be *above* the axis of x, and consequently they will contain the area $(\frac{16}{3}\sqrt{3}-9)\pi a^2$, as found by Prof. Johnson. On the other hand, if we consider only the triangles that can be drawn on one side of the hypothenuse, and describe the squares on both sides of the legs, we obtain two branches respectively equivalent to those found by Prof. Johnson, but one lying above and the other below the axis of x, and consequently enclosing an area $=\frac{16}{9}\sqrt{3}\,.\,\pi a^2-4a^2$, as given by us at page 156.

Both branches as thus found are represented by equation (4), page 155, but one, the outer branch, is excluded by the wording of the question, and hence Messrs. Seitz, Heaton and Baker, by describing the squares only on the outside of the legs, find two equal branches, one above and the other below the axis of x, which enclose an area $=(9-\frac{32}{9}\sqrt{3})\pi a^2-8a^2$.